

Spin-one color superconductors: collective modes and effective Lagrangian

Jin-yi Pang^a, Tomáš Brauner^{b,1}, Qun Wang^a

^a*Interdisciplinary Center for Theoretical Study and Department of Modern Physics, University of Science and Technology of China, Anhui 230026, People's Republic of China*

^b*Faculty of Physics, University of Bielefeld, D-33501 Bielefeld, Germany*

Abstract

We investigate the collective excitations in spin-one color superconductors. We classify the Nambu–Goldstone modes by the pattern of spontaneous symmetry breaking, and then use the Ginzburg–Landau theory to derive their dispersion relations. These soft modes play an important role for the low-energy dynamics of the system such as the transport phenomena and hence are relevant for late-stage evolution of neutron stars. In the case of the color-spin-locking phase, we use a functional technique to obtain the low-energy effective action for the physical Nambu–Goldstone bosons that survive after gauging the color symmetry.

Keywords: Color superconductivity, Spontaneous symmetry breaking, Nambu–Goldstone bosons

1. Introduction

In the phase diagram of quantum chromodynamics (QCD), cold quark matter is expected to deconfine at high density and be in a color superconducting state (see [1–4] for recent reviews). Understanding the properties of matter under such extreme conditions is, apart from being an integral part of the quest for the fundamental laws of nature, relevant for the astrophysics of compact stellar objects. The density in their cores is presumably high enough to form deconfined quark matter.

Since the energy scale for strong interaction in astrophysical processes mentioned above is of order 100 MeV, one can safely neglect the heavy quark flavors and only consider the three lightest ones, up (u), down (d), and strange (s). At densities so high that the masses of all light quarks can be neglected, the ground state of three-flavor quark matter is known to be the color-flavor-locking (CFL) state. However, as the chemical potential drops to the range interesting for astrophysical applications, the strange quark mass is not negligible and starts to play an important role. It reduces the Fermi sea of s quarks, leading to an excess electric charge which must in turn be compensated by an imbalance between the u and d quarks. As a consequence, the highly symmetric CFL state feels stress and can give way to other pairing patterns [5].

If the stress on the CFL pairing induced by the strange quark mass is too high only the u and d quarks pair, which is usually denoted as the 2SC phase. However, as explained above, the energy gain from the 2SC pairing is reduced by the requirement of electric charge neutrality. It may then happen that the mismatch between the Fermi levels is so large that pairing between quarks of different flavors is completely ruled out. In such a case, one is still left with the possibility of pairing quarks of the *same* flavor. Nevertheless, since the QCD-induced effective quark–quark interaction is attractive in the color-antisymmetric channel, this requires the wave function of the Cooper pair to be symmetric in Lorentz indices by the Pauli principle, i.e. to carry nonzero spin. While spin one is the simplest possibility, the ground state will in general be a mixture of all partial waves with an odd spin [6].

There are several situations in which spin-one pairing may occur. Apart from the pairing of quarks of a single flavor considered originally in Refs. [7–10], there is another possibility of pairing of quarks of the same *color* [11].

Email addresses: pangjin@mail.ustc.edu.cn (Jin-yi Pang), tbrauner@physik.uni-bielefeld.de (Tomáš Brauner), qunwang@ustc.edu.cn (Qun Wang)

¹On leave from Department of Theoretical Physics, Nuclear Physics Institute ASCR, CZ-25068 Řež, Czech Republic

This naturally appears in the 2SC phase where only quarks of two colors are involved in the pairing and the third one is left over. We will use this proposal as a warm-up exercise in Sec. 2. However, in the rest of the paper, we will focus on the more likely pattern of pairing of quarks of the same flavor. Even then one can distinguish different scenarios. When cross-flavor pairing is completely prohibited, all three flavors will pair in some of the spin-one states. On the other hand, only s quarks may undergo spin-one pairing as a complement to the 2SC state. We will have in mind mostly the first case, since in the latter the presence of the color-asymmetric 2SC state can modify the ground state of the spin-one phase [12].

The classification of the phases of a single-flavor spin-one color superconductor was worked out in Refs. [13, 14]. The physical properties of the so-called inert spin-one phases were investigated in Refs. [15–17]. Astrophysical implications of the presence of a spin-one phase were discussed in Refs. [18–25]. An alternative approach to spin-one color superconductors, based on the Schwinger–Dyson equations, was taken in Ref. [26]. For some recent references on the topic see Refs. [27–29].

It was shown in Ref. [14] that as a consequence of weak interactions, the true ground state of spin-one color superconductors is an inhomogeneous state with helical ordering, similar to what happens in non-center-symmetric ferromagnets [30, 31]. In the present paper this additional structure will not, for the sake of simplicity, be considered. This may be understood as restricting to length scales much larger than the average distance between quarks, yet much smaller than the wavelength of the helix.

The plan of the paper is as follows. In Sec. 2 we shall investigate the collective modes in a single-color spin-one superconductor [11]. This elucidates some of the peculiarities of spontaneous breaking of spacetime symmetry [32] without the technical complications brought by a complex matrix order parameter. In particular, we will show an explicit example of a phase in which the number of Nambu–Goldstone (NG) bosons is smaller than the number of broken symmetry generators [33] (see Ref. [34] for a recent review including more references). Also, it will be demonstrated that due to the fact that the order parameter breaks rotational symmetry, the division of the NG modes into multiplets of the unbroken symmetry holds only in the long-wavelength limit. Nonzero momentum of the mode breaks the remaining symmetry and results in further splitting of the multiplets.

Section 3 is the main content of the paper. We analyze the spectrum of collective modes in a spin-one color superconductor, elaborating on the classification of the NG modes suggested in our previous paper [35]. For simplicity, we consider quark matter composed of one quark flavor only. This assumption is released in Sec. 4 where we concentrate on the color-spin-locking (CSL) phase in electrically neutral three-flavor quark matter. Using a generalization of a trick due to Son [36], we construct the low-energy effective Lagrangian for the physical NG bosons that remain in the spectrum after gauging the color and electromagnetic sectors of the symmetry group. Finally, in Sec. 5 we summarize and make conclusions.

2. Single-color spin-one superconductor

The pairing pattern considered here was suggested in Ref. [11]. Since it complements the standard 2SC state, it involves quarks of the two lightest flavors and one color. They pair in a flavor-singlet state, and the wave function must therefore be symmetric in the Lorentz indices. In the simplest case of spin one, the order parameter is a flavor singlet and spin triplet, and thus is represented by a complex vector of the $SO(3)$ rotational group.

We will analyze the excitation spectrum using a Lagrangian which can be regarded as the time-dependent Ginzburg–Landau (GL) theory. We will demand that the Lagrangian has rotational as well as $U(1)$ phase invariance, corresponding to the conservation of particle number, and preserves parity. The most general Lagrangian for a complex vector field ϕ that has the required symmetries and includes operators of canonical dimension up to four, reads

$$\mathcal{L} = ic_1 \phi^\dagger \cdot \partial_0 \phi + c_2 \partial_0 \phi^\dagger \cdot \partial_0 \phi - a_1 \partial_i \phi^\dagger \cdot \partial_i \phi - a_2 |\partial \cdot \phi|^2 - b \phi^\dagger \cdot \phi - d_1 (\phi^\dagger \cdot \phi)^2 - d_2 |\phi \cdot \phi|^2, \quad (1)$$

where the spatial vectors are in boldface and their inner product is indicated by a dot; we will use this convention throughout the paper. One should keep in mind that by rescaling the field appropriately, one can get rid of one of the unknown coefficients, say, c_1 . The static part of this Lagrangian was investigated in [11], so we just summarize the results here. First of all, boundedness of the potential from below demands that $d_1 > 0$ and $d_1 + d_2 > 0$. Once $b < 0$, the scalar field condenses, that is, develops nonzero vacuum expectation value. For $d_2 < 0$ the ground state has the form $\phi_0 = v(0, 0, 1)^T$, where $v^2 = -\frac{b}{2(d_1+d_2)}$. We will call this the *polar phase* in analogy with the three-color

spin-one color superconductor. For $d_2 > 0$ the ground state can be chosen as $\phi_0 = \frac{v}{\sqrt{2}}(1, i, 0)^T$ with $v^2 = -\frac{b}{2d_1}$. This is analogous to the *A-phase*.

In both phases, the $\text{SO}(3) \times \text{U}(1)$ global symmetry is broken to a $\text{U}(1)'$ subgroup. In the polar phase, this simply corresponds to rotations in the (ϕ_1, ϕ_2) plane, while in the A-phase, one has to change the overall phase simultaneously with the rotation to keep the order parameter unchanged. In the polar phase, all Noether charges of the global $\text{SO}(3)$ symmetry are zero in the ground state. As a consequence, we expect to find three NG bosons with linear dispersion relations at low momentum, associated with the three spontaneously broken generators. On the other hand, in the A-state the third component of spin, represented in the ϕ -space by the matrix $(J_i)_{jk} = -i\epsilon_{ijk}$, has nonzero density. Since J_3 belongs to a non-Abelian symmetry group, the NG boson counting will be modified [33, 37]. We expect one type-I NG boson with a linear dispersion relation and one type-II NG boson with a quadratic dispersion relation. The latter represents a circularly polarized spin wave, very much like in nonrelativistic ferromagnets. We will now see how these predictions, based on general properties of spontaneous symmetry breaking in many-body systems [34], are verified in an explicit calculation.

2.1. Polar phase

The unbroken $\text{U}(1)' \sim \text{SO}(2)$ subgroup is generated by J_3 while the broken generators are $J_{1,2}$ and the generator of phase transformations. The six-(real-)component complex field ϕ can therefore be parameterized as

$$\phi = e^{\frac{i}{v}\theta} e^{\frac{i}{v}\pi J} (\phi_0 + \mathbf{H} + i\chi), \quad (2)$$

where θ and $\pi \equiv (\pi_1, \pi_2, 0)^T$ are the NG modes, whereas the remaining degrees of freedom, $\chi = (-\chi_2, \chi_1, 0)^T$ and $\mathbf{H} = (0, 0, H)^T$, are anticipated to excite massive states in the spectrum. The parameterization of the vector χ is just for convenience: to first order in the fields the vector $i\chi$ is then “ i times π ”. Plugging this parameterization into the Lagrangian (1), expanding up to second order in the fields, and throwing away all total derivatives, we end up with the bilinear Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{bilin}} = & -2c_1(H\partial_0\theta - \chi \cdot \partial_0\pi) + \sum_{\mu} \left(\begin{matrix} \mu=0 : c_2 \\ \mu=i : -a_1 \end{matrix} \right) [(\partial_\mu\theta)^2 + (\partial_\mu\pi)^2 + (\partial_\mu H)^2 + (\partial_\mu\chi)^2] \\ & -a_2 [(\partial_3 H - \text{rot}_\perp \pi)^2 + (\partial_3 \theta - \text{rot}_\perp \chi)^2] - 4(d_1 + d_2)v^2 H^2 + 4d_2 v^2 \chi^2, \end{aligned} \quad (3)$$

where the planar rotation operator is defined as $\text{rot}_\perp \varphi \equiv \partial_1 \varphi_2 - \partial_2 \varphi_1$.

Note that only the H and χ modes have mass terms as expected, moreover, the χ mass vanishes for $d_2 = 0$. This signals the instability associated with the phase transition from the polar to the A-phase. On the other hand, the H mass term can never change sign since $d_1 + d_2 > 0$ is required by the stability of the potential. All six modes are in general mixed by the derivative terms so that a direct diagonalization of the Lagrangian would have to be done numerically. However, given that spatial rotations are spontaneously broken to the $\text{SO}(2)$ subgroup, we can investigate separately excitations along the symmetry-breaking (ϕ_3) axis, and in the “transverse” (ϕ_1, ϕ_2) plane. The analysis then simplifies considerably and essentially reduces to mixing of two fields at a time. This is discussed in general in Appendix A.

Let us first assume that the fields depend just on the time and the third coordinate. The complicated mixing in the a_2 term then reduces to a mere modification of the phase velocities of the H and θ fields. Only the singlets H and θ , and vectors χ and π now mix. The dispersion relations are found using Eqs. (A.2) and (A.3). In the (H, θ) sector they read

$$E^2 = \frac{1}{c_2^2} [4v^2(d_1 + d_2)c_2 + c_1^2] \quad (\text{massive mode}), \quad E^2 = \frac{4v^2(d_1 + d_2)(a_1 + a_2)}{4v^2(d_1 + d_2)c_2 + c_1^2} k_3^2 \quad (\text{NG mode}), \quad (4)$$

to the lowest order in the momentum expansion, while in the (χ, π) sector they are

$$E^2 = \frac{1}{c_2^2} (-4v^2 d_2 c_2 + c_1^2) \quad (\text{massive modes}), \quad E^2 = \frac{-4v^2 d_2 a_1}{-4v^2 d_2 c_2 + c_1^2} k_3^2 \quad (\text{NG modes}). \quad (5)$$

Next we assume that the fields depend just on time and the first two coordinates. The H and θ modes then decouple from others, giving rise to dispersion relations

$$E^2 = \frac{1}{c_2^2} [4v^2(d_1 + d_2)c_2 + c_1^2] \quad (\text{massive mode}), \quad E^2 = \frac{4v^2(d_1 + d_2)a_1}{4v^2(d_1 + d_2)c_2 + c_1^2} k_\perp^2 \quad (\text{NG mode}), \quad (6)$$

where the subscript “ \perp ” indicates that the mode is transverse. On the other hand, the a_2 term still seemingly mixes the components of the χ and π vectors. However, redefining them to $\tilde{\chi} = (\chi_1, \chi_2, 0)^T$ and $\tilde{\pi} = (\pi_2, -\pi_1, 0)^T$, one gets $\text{rot}_\perp \chi = \partial \cdot \tilde{\chi}$ and $\text{rot}_\perp \pi = \partial \cdot \tilde{\pi}$, while the c_1 term is not affected since $\tilde{\chi} \cdot \tilde{\pi} = \chi \cdot \pi$. The excitations in the (χ, π) sector then further split into two branches. Modes for which $\tilde{\chi}$ and $\tilde{\pi}$ are parallel to the momentum behave as longitudinal and their dispersion relations are given by

$$E^2 = \frac{1}{c_2^2}(-4v^2 d_2 c_2 + c_1^2) \quad (\text{massive mode}), \quad E^2 = \frac{-4v^2 d_2(a_1 + a_2)}{-4v^2 d_2 c_2 + c_1^2} k_\perp^2 \quad (\text{NG mode}). \quad (7)$$

For $\tilde{\chi}$ and $\tilde{\pi}$ modes perpendicular to the momentum, the a_2 term vanishes and the dispersion relations are identical to those found in Eq. (5).

Note that the masses of the massive modes naturally do not depend on the chosen direction of momentum, as could have been expected. Also, it is now obvious that nonzero momentum lifts degeneracy based on the symmetry of the ground state. Based on the unbroken $\text{SO}(2)$ symmetry we would have expected both χ and π to transform as vectors whereas H and θ as singlets. This is indeed the case for the longitudinal excitations. On the other hand, transverse momentum breaks the remaining $\text{SO}(2)$ symmetry so that the π_1, π_2 NG modes are no longer degenerate. A similar remark applies to all other results in this and the following section.

2.2. A-phase

The ground state is an eigenvector of J_3 , that is, $J_3 \phi_0 = \phi_0$. This implies that the unbroken subgroup is generated by the combination $\frac{1}{2}(\mathbb{1} - J_3)$. The broken generators can be conveniently chosen as J_1, J_2 , and $\frac{1}{2}(\mathbb{1} + J_3)$. So ϕ can be parameterized as

$$\phi = e^{i\pi J'} \left(\phi_0 + \frac{H}{v} \phi_0 + \chi \phi_1 \right), \quad (8)$$

where $J' = (J_1, J_2, \frac{\mathbb{1} + J_3}{2})$, $\pi = (\pi_1, \pi_2, \pi_3)^T$, and $\phi_1 = \frac{1}{\sqrt{2}}(1, -i, 0)^T$ is a vector perpendicular to ϕ_0 . The real field H and the complex field χ describe the non-NG modes. When used in the Lagrangian (1), this parameterization yields the following bilinear terms

$$\begin{aligned} \mathcal{L}_{\text{bilin}} = & ic_1 \left[\chi^* \partial_0 \chi + 2iH \partial_0 \pi_3 + \frac{i}{2}(\pi_1 \partial_0 \pi_2 - \pi_2 \partial_0 \pi_1) \right] \\ & + \sum_{\mu} \left(\begin{array}{l} \mu = 0 : c_2 \\ \mu = i : -a_1 \end{array} \right) \left[\frac{1}{2}(\partial_\mu \pi_{1,2})^2 + (\partial_\mu \pi_3)^2 + (\partial_\mu H)^2 + |\partial_\mu \chi|^2 \right] \\ & - \frac{a_2}{2} \left[(\partial_3 \pi_2 - \partial_2 \pi_3 + \partial_1 \chi_1 + \partial_2 \chi_2 + \partial_1 H)^2 + (\partial_3 \pi_1 - \partial_1 \pi_3 + \partial_2 \chi_1 - \partial_1 \chi_2 - \partial_2 H)^2 \right] \\ & - 4d_1 v^2 H^2 - 4d_2 v^2 |\chi|^2, \end{aligned} \quad (9)$$

where $\chi_{1,2}$ are the real and imaginary parts of χ . Again, thanks to the exponential parameterization the NG fields are explicitly eliminated from the static part of the Lagrangian. Also, the χ mass term is seen to vanish at the transition to the polar phase, i.e., $d_2 = 0$.

To determine the excitation spectrum, we will first assume that the fields depend only on time and the third coordinate. The fields then fall into three sectors that do not mix with each other. The field χ carries nonzero charge of the unbroken $\text{U}(1)'$ symmetry and describes a particle–antiparticle pair with masses

$$E = \frac{1}{2c_2} \left(\pm c_1 + \sqrt{16v^2 d_2 c_2 + c_1^2} \right). \quad (10)$$

The H and π_3 modes give rise, similarly to the polar phase, to one massive and one NG state of type-I,

$$E^2 = \frac{1}{c_2^2}(4v^2 d_1 c_2 + c_1^2) \quad (\text{massive mode}), \quad E^2 = \frac{4v^2 d_1 a_1}{4v^2 d_1 c_2 + c_1^2} k_3^2 \quad (\text{NG mode}). \quad (11)$$

On the contrary, the (π_1, π_2) sector, which naively contains two NG bosons, only produces one NG particle of type-II, in agreement with the general discussion in [37],

$$E = \frac{|c_1|}{c_2} \quad (\text{massive mode}), \quad E = \frac{a_1 + a_2}{|c_1|} k_3^2 \quad (\text{type-II NG mode}). \quad (12)$$

Second, we will investigate the transverse excitations. The masses of the particles excited by χ_1, χ_2 are still given by Eq. (10); the anisotropy brought by nonzero momentum does not appear in this lowest-order term in the dispersion relation. For the (H, π_3) fields we obtain the following dispersions,

$$E^2 = \frac{1}{c_2^2} (4v^2 d_1 c_2 + c_1^2) \quad (\text{massive mode}), \quad E^2 = \frac{4v^2 d_1 (a_1 + \frac{a_2}{2})}{4v^2 d_1 c_2 + c_1^2} k_\perp^2 \quad (\text{type-II NG mode}). \quad (13)$$

The π_1 and π_2 modes trivially decouple and give rise to the dispersions

$$E = \frac{|c_1|}{c_2} \quad (\text{massive mode}), \quad E = \frac{a_1}{|c_1|} k_\perp^2 \quad (\text{type-II NG mode}). \quad (14)$$

Before we conclude the section we remark that the calculation of the dispersion relations is complicated by the fact that spacetime symmetry is spontaneously broken. However, the basic anticipated features of the NG spectrum are preserved. In both phases three generators are spontaneously broken. In the polar phase, they give rise to three type-I NG bosons with linear dispersion relation at low momentum. In the A-phase, the ground state carries nonzero spin density, therefore the three broken generators produce one type-I NG boson with linear dispersion and one type-II NG boson with quadratic dispersion at low momentum.

Finally, at the transition point between the two phases $d_2 = 0$, the static part of the Lagrangian has an extended $\text{SO}(6)$ symmetry under which the polar and A-phase order parameters are degenerate. Five of its generators are broken, leaving an $\text{SO}(5)$ invariant subgroup. This extended symmetry is also reflected in the NG spectrum. For instance, in the polar phase the phase velocities of two of the NG bosons go to zero so that their dispersions are quadratic. This is in accordance with the general Nielsen–Chadha counting rule [33]. However, note that this extended symmetry is explicitly broken by the c_1 and a_2 terms in the Lagrangian (1). The extra NG bosons are therefore only present in the classical theory, they will acquire nonzero masses via radiative corrections [38].

3. Single-flavor spin-one color superconductor

In this section we will study the spin-one color superconductor that involves pairing of quarks of three colors but a single flavor. The diquark condensate or the order parameter Δ is then a color antitriplet and spin triplet, so it is a 3×3 complex matrix and transforms as

$$\Delta \rightarrow U \Delta R \quad (15)$$

where $U = \exp(i\theta_a \lambda_a) \in \text{U}(3)_L = \text{SU}(3)_c \times \text{U}(1)_B$ and $R = \exp(i\alpha_i J_i) \in \text{SO}(3)_R$ are transformation matrices. Here λ_a are eight Gell-Mann matrices and $\lambda_0 \equiv \sqrt{\frac{2}{3}} \mathbb{1}$, $(J_i)_{jk} = -i\epsilon_{ijk}$ are generators of $\text{SO}(3)_R$, θ_a ($a = 0, \dots, 8$) and α_i ($i = 1, 2, 3$) are rotation angles in $\text{U}(3)_L$ and $\text{SO}(3)_R$ group space. There are 18 real parameters in Δ , among which 12 parameters are carried by the $\text{U}(3)_L \times \text{SO}(3)_R$ transformation making a 12-dimensional degenerate vacuum manifold. Then Δ can be parameterized by the remaining 6 real parameters which characterize different vacuum states [14],

$$\Delta = \begin{pmatrix} \Delta_1 & i\delta_3 & -i\delta_2 \\ -i\delta_3 & \Delta_2 & i\delta_1 \\ i\delta_2 & -i\delta_1 & \Delta_3 \end{pmatrix}. \quad (16)$$

3.1. Ginzburg–Landau free energy and ground states

The GL analysis is similar to that for superfluid Helium 3 [39]. Up to fourth order in Δ and two derivatives, the most general $U(3)_L \times SO(3)_R$ and parity invariant Ginzburg–Landau free energy density functional can be written as

$$\begin{aligned} \mathcal{F}[\Delta] = & a_1 \text{Tr}(\partial_i \Delta \partial_i \Delta^\dagger) + a_2 (\partial_i \Delta_{ai})(\partial_j \Delta_{aj}^*) + b \text{Tr}(\Delta \Delta^\dagger) \\ & + d_1 [\text{Tr}(\Delta \Delta^\dagger)]^2 + d_2 \text{Tr}(\Delta \Delta^\dagger \Delta \Delta^\dagger) + d_3 \text{Tr}[\Delta \Delta^T (\Delta \Delta^T)^\dagger]. \end{aligned} \quad (17)$$

The time-dependent GL functional, or Lagrangian, is then in general written as

$$\mathcal{L} = ic_1 \text{Tr}[\Delta^\dagger \partial_0 \Delta] + c_2 \text{Tr}[(\partial_0 \Delta^\dagger)(\partial_0 \Delta)] - \mathcal{F}[\Delta], \quad (18)$$

see Appendix B for more details and the relation between the coefficients c_1, c_2 . The ground state is found by minimizing $\mathcal{F}[\Delta]$. The sign of b determines whether the order parameter is zero or nonzero. Hereafter we will assume that $b < 0$. The phase structure, or orientation in the field space, of Δ_0 depends on d_2 and d_3 . The magnitude of the condensate, $v = \sqrt{\text{Tr}(\Delta \Delta^\dagger)}$, is given by $v = \sqrt{-b/(2\bar{d})}$ with $\bar{d} = d_1 + f(d_2, d_3)$ (f is a function of d_2 and d_3 that is specific to a particular phase) and the vacuum energy is $E_{\text{vac}} = -b^2/(4\bar{d})$. The boundedness of the potential from below demands that $\bar{d} > 0$, which constrains the possible values of d_1 for a given phase. The symmetry of the problem allows for altogether 8 inequivalent states with different patterns of spontaneous breaking of the *continuous* symmetry. However, only the following four of them occupy a part of the phase diagram [14],

$$\Delta_{\text{CSL}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Delta_{\text{polar}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Delta_{\text{A}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & i & 0 \end{pmatrix}, \quad \Delta_{\text{E}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta \\ \alpha & i\alpha & 0 \end{pmatrix}, \quad (19)$$

where $\alpha = \sqrt{(d_2 + d_3)/[2(2d_2 + d_3)]}$ and $\beta = \sqrt{d_2/(2d_2 + d_3)}$. The pattern of spontaneous symmetry breaking determines the low-energy spectrum of the system, i.e., the NG bosons. While some of the NG bosons are associated with the generators of the color $SU(3)_c$ group and are thus eventually absorbed in gluons via the Higgs–Anderson mechanism, those stemming from spontaneous breaking of baryon number or rotation symmetry remain in the spectrum as physical soft modes. As we will now see, some of the phases exhibit the unusual type-II NG bosons, in accordance with general properties of spontaneously broken symmetries in quantum many-body systems [33, 34]. We did not solve the fully coupled equations of motion for the fields depending simultaneously on all coordinates, so the dispersion relations shown in the following should be understood as combinations of separate formulas for the “longitudinal” and “transverse” excitations.

3.2. CSL phase

When $d_2 + d_3 > 0$ and $d_2 > d_3$, the ground state is the CSL phase whose order parameter is given in Eq. (19). Also, $\bar{d} = d_1 + \frac{d_2 + d_3}{3}$. In the CSL phase the spin and color are coupled in the pairing, so the symmetry breaking pattern is $U(3)_L \times SO(3)_R \rightarrow SO(3)_V$. The generators of the unbroken symmetry $SO(3)_V$ are $\sqrt{\frac{1}{2}}(\lambda_7 \otimes \mathbb{1} + \mathbb{1} \otimes J_1)$, $\sqrt{\frac{1}{2}}(-\lambda_5 \otimes \mathbb{1} + \mathbb{1} \otimes J_2)$, $\sqrt{\frac{1}{2}}(\lambda_2 \otimes \mathbb{1} + \mathbb{1} \otimes J_3)$. There are 9 broken generators leading to 9 NG bosons as follows,

- $\lambda_0 \otimes \mathbb{1}$. Type-I NG singlet, $E^2 \sim (a_1 + a_2)k^2$.
- $\sqrt{\frac{1}{2}}(\lambda_7 \otimes \mathbb{1} - \mathbb{1} \otimes J_1)$, $\sqrt{\frac{1}{2}}(-\lambda_5 \otimes \mathbb{1} - \mathbb{1} \otimes J_2)$, $\sqrt{\frac{1}{2}}(\lambda_2 \otimes \mathbb{1} - \mathbb{1} \otimes J_3)$. Type-I NG triplet, $E^2 \sim (a_1 + a_2)k^2$.
- $\lambda_\alpha \otimes \mathbb{1}$, $\alpha = 1, 3, 4, 6, 8$. Type-I NG 5-plet, $E^2 \sim (a_1 + a_2)k^2$.

The details of the above modes are given in Appendix B.1. After gauging the color symmetry, the type-I NG 5-plet is absorbed by gluons, and we are left with the singlet as the only physical NG boson, stemming from the spontaneous breaking of the $U(1)_B$ symmetry.

3.3. Polar phase

When $d_3 < 0$ and $d_2 + d_3 < 0$, the ground state is the polar phase whose order parameter is shown in Eq. (19), and $\bar{d} = d_1 + d_2 + d_3$. The symmetry breaking pattern is $U(3)_L \times SO(3)_R \rightarrow U(2)_L \times SO(2)_R$. The unbroken symmetry is generated by $\lambda_{1,2,3} \otimes \mathbb{1}$, $\mathcal{P}_{12} \otimes \mathbb{1}$ and $\mathbb{1} \otimes J_3$, where $\mathcal{P}_{12} = \frac{1}{\sqrt{3}}(\sqrt{2}\lambda_0 + \lambda_8) = \text{diag}(1, 1, 0)$ is the projector onto the first two colors. The diquark spin is polarized to one direction. There are 7 broken generators which, however, give rise only to 5 NG bosons, organized in the following multiplets,

- $\sqrt{2}\mathcal{P}_3 \otimes \mathbb{1}$, where $\mathcal{P}_3 = \frac{1}{\sqrt{6}}(\lambda_0 - \sqrt{2}\lambda_8) = \text{diag}(0, 0, 1)$ is the projector onto the third color. Type-I NG singlet, $E^2 \sim a_1 k_\perp^2 + (a_1 + a_2)k_3^2$.
- $\mathbb{1} \otimes J_j$, $j = 1, 2$. Type-I NG doublet, $E^2 \sim (a_1 + a_2)k_\perp^2 + a_1 k_3^2$.
- $\lambda_\alpha \otimes \mathbb{1}$, $\alpha = 4, 5, 6, 7$. Type-II NG doublet, $E^2 \sim a_1^2 k_\perp^4 + (a_1 + a_2)^2 k_3^4$.

The presence of type-II NG bosons is due to nonzero color density of the polar ground state. The details of the above modes are given in Appendix B.2. After gauging the color symmetry, the type-I NG singlet and the type-II NG doublet are absorbed by gluons, only the type-I NG doublet survives, corresponding to two linearly polarized spin waves.

3.4. A-phase

When $d_3 > 0$ and $d_2 < 0$, the ground state is the A-phase whose order parameter is shown in Eq. (19), and $\bar{d} = d_1 + d_2$. The symmetry breaking pattern is $U(3)_L \times SO(3)_R \rightarrow U(2)_L \times SO(2)_V$. The residual symmetry is generated by $\lambda_{1,2,3} \otimes \mathbb{1}$, $\mathcal{P}_{12} \otimes \mathbb{1}$ and $\sqrt{\frac{2}{3}}(\mathcal{P}_3 \otimes \mathbb{1} - \mathbb{1} \otimes J_3)$. Unlike in the polar phase, the diquark spin is now circularly polarized. Among 7 broken generators, there is only one giving rise to a type-I NG mode,

- $\sqrt{\frac{2}{3}}(\mathcal{P}_3 \otimes \mathbb{1} + \mathbb{1} \otimes J_3)$. Type-I NG singlet, $E^2 \sim (a_1 + a_2)k_\perp^2 + a_1 k_3^2$.

The rest 6 generators produce only 3 type-II NG bosons due to non-zero color and spin density of the A-phase vacuum,

- $\lambda_\alpha \otimes \mathbb{1}$, $\alpha = 4, 5, 6, 7$. Type-II NG doublet, $E^2 \sim (a_1 + a_2)^2 k_\perp^4 + a_1^2 k_3^4$.
- $\mathbb{1} \otimes \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)$. Type-II NG singlet, $E^2 \sim a_1^2 k_\perp^4 + (a_1 + a_2)^2 k_3^4$.

See Appendix B.3 for details on the above modes. After gauging the color symmetry, the type-I NG singlet and the type-II NG doublet are absorbed by gluons and leave the type-II NG singlet giving a circular spin wave.

3.5. ε -phase

When $d_3 > d_2 > 0$, the ground state is the ε phase, see Eq. (19) for its order parameter; in this case $\bar{d} = d_1 + \frac{d_2(d_2+d_3)}{2d_2+d_3}$. The symmetry breaking pattern is $U(3)_L \times SO(3)_R \rightarrow U(1)_L \times SO(2)_V$. The spin of the second diquark color is longitudinally polarized, while that of third color is circularly polarized. Like in the A-phase, the circularly polarized spin produces an $SO(2)_V$ unbroken symmetry. The unbroken symmetry is generated by $\sqrt{2}\mathcal{P}_1 \otimes \mathbb{1}$ and $\sqrt{\frac{2}{3}}(\mathcal{P}_3 \otimes \mathbb{1} - \mathbb{1} \otimes J_3)$, where $\mathcal{P}_1 = \text{diag}(1, 0, 0)$ is the projector onto the first color. Out of the 10 broken generators only two correspond to type-I NG modes:

- $\sqrt{2}\mathcal{P}_2 \otimes \mathbb{1}$, where $\mathcal{P}_2 = \text{diag}(0, 1, 0)$ is the projector onto the second color. Type-I NG singlet, $E^2 \sim a_1 k_\perp^2 + (a_1 + a_2)k_3^2$.
- $\sqrt{\frac{2}{3}}(\mathcal{P}_3 \otimes \mathbb{1} + \mathbb{1} \otimes J_3)$. Type-I NG singlet, $E^2 \sim (a_1 + a_2)k_\perp^2 + a_1 k_3^2$.

The remaining 8 generators give rise to 4 type-II NG modes due to non-zero color and spin density of the ε vacuum,

- $\frac{1}{\sqrt{2}}(\lambda_1 \pm i\lambda_2) \otimes \mathbb{1}$. Type-II NG singlet, $E^2 \sim a_1^2 k_\perp^4 + (a_1 + a_2)^2 k_3^4$.

- $\frac{1}{\sqrt{2}}(\lambda_{4,6} \pm i\lambda_{5,7}) \otimes \mathbb{1}$. Type-II NG doublet, $E^2 \sim (a_1 + a_2)^2 k_\perp^4 + a_1^2 k_3^4$.
- $\mathbb{1} \otimes \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)$. Type-II NG singlet, $E^2 \sim a_1^2 k_\perp^4 + (a_1 + a_2)^2 k_3^4$.

See Appendix B.4 for details on the above modes. After gauging the color symmetry, only the last type-II NG singlet survives, corresponding to a circularly polarized spin wave.

4. Low energy effective field theory for the CSL phase

The CSL phase plays in many respects a distinguished role among all spin-one phases investigated in the preceding section. First, it is the ground state of one-flavor quark matter in the limit of very high chemical potential. Second, it is isotropic and involves democratically all quark colors. Consequently, under some additional assumptions on the spin-orbital structure of the order parameter, all quarks are gapped and the low-energy spectrum is solely determined by the NG bosons of the spontaneously broken symmetry. Moreover, the rotational symmetry is unbroken whereas the NG bosons associated with the color symmetry are absorbed into gluons once this is gauged. Therefore the low-energy physics of the CSL phase will be governed by the NG bosons of the global *Abelian* symmetry. In this section, we will employ a technique of Son [36], which was previously used to study the transport properties of the CFL phase [40].

4.1. Global symmetry and NG bosons

Based on the argument in the previous paragraph, one would naively conclude that in the one-flavor CSL phase, there is exactly one physical NG boson, stemming from the spontaneous breaking of the global U(1) symmetry. However, this is only true provided the quarks do not carry any other gauge degrees of freedom apart from the color ones. This is certainly a very rough approximation considering that the spin-one phases are only likely to occur in the phase diagram in the region where strange quark mass is large and electric charge neutrality effects play an important role.

As already discussed in the Introduction, there are two realistic scenarios for spin-one pairing to occur in three-flavor quark matter [1]. (i) The u and d quarks are paired in the 2SC phase and only the s quarks are left and undergo the single-flavor spin-one pairing. (ii) Cross-flavor pairing is completely prohibited by large Fermi level mismatch and all three quark flavors undergo single-flavor pairing. While in the case (ii) the most favored pairing pattern will be CSL, in the case (i) it may not be so. The reason is that the 2SC phase is not color neutral; to compensate for its color charge, a color chemical potential must be introduced. This in turn breaks the exact color symmetry of the s quark sector. In addition, the mismatch between different colors may favor other pairing patterns such as the polar one [12].

For these reasons, we will only consider the scenario (ii) which is theoretically clean; each flavor feels an exact color SU(3) symmetry and features the same symmetry breaking pattern. The question then is: how many NG bosons are there? As mentioned above, the NG bosons of the spontaneously broken color symmetry will be absorbed into gluons. In addition, there is a global Abelian symmetry, $G = U(1)_u \times U(1)_d \times U(1)_s$, corresponding simply to separate conservation of the flavor quark numbers. This will give rise to three NG bosons. However, a subgroup of G , given by the electromagnetic $U(1)_Q$, is gauged and the associated NG boson will be eaten by the photon, making it massive and thus giving rise to the Meissner effect. Therefore, there will be only two physical NG bosons left.

One should note that in special limits, the flavor symmetry can be actually larger than G . For example, assuming that the quark masses satisfy $0 \neq m_u = m_d \neq m_s$, the flavor symmetry will be $\tilde{G} = SU(2)_V \times U(1)_{u+d} \times U(1)_s$. However, the non-Abelian $SU(2)_V$ group will be explicitly broken down to $U(1)_{I_3}$, generated by the third component of isospin, by the electric charge chemical potential necessary to maintain electric charge neutrality. The remaining exact symmetry group is isomorphic to G . The very fact that u and d quarks are actually light does not play a role since the pions will presumably still be heavier than the CSL gap so that they will not enter the low-energy effective field theory whose validity is limited by the scale of the gap.

4.2. General method to construct the effective Lagrangian

A general method to construct the low-energy effective action for the NG boson of a spontaneously broken $U(1)$ symmetry at zero temperature was proposed by Son based on a functional technique [36]. Starting with the equation of state, $P(\mu)$, the effective action for the NG field φ to the lowest order in derivatives reads

$$\Gamma[\mu, \varphi] = \int d^4x P\left(\sqrt{(\partial_0\varphi - \mu)^2 - (\partial_i\varphi)^2}\right). \quad (20)$$

Several remarks are in order here. First, this is a fully *quantum* effective action, that is, it should be used strictly at tree level. All loop effects are included in the couplings of the action. Second, after expansion in powers of φ , the action contains only terms with the same number of derivatives as is the power of φ . This is the leading-order term in the derivative expansion for a given power of φ . Third, the derivation relies heavily on the fact that μ is the only source of Lorentz violation in the theory, since then the full dependence on the NG field can be reconstructed from the dependence of the pressure on the chemical potential using the fact that the NG field and the chemical potential only appear in the effective action in the combination $\partial_\nu\varphi - \delta_{\nu 0}\mu$.

Once we know the equation of state, we plug it into Eq. (20) and expand in powers of φ to obtain both the dispersion relation of the NG boson and its self-interactions. In extremely dense quark matter one can, thanks to asymptotic freedom, take as a reasonable starting point the equation of state for a free massless Fermi gas $P_0(\mu) = N_c\mu^4/(12\pi^2)$, for a single quark flavor [36, 40]. The effect of pairing on the equation of state can be neglected since the CSL gap is numerically very small. However, one may be interested in corrections due to nonzero quark mass since it is exactly the strange quark mass that opens the way to the spin-one phases in the phase diagram. To that end, one needs to know the equation of state of a massive Fermi gas,

$$\frac{P_m(\mu)}{N_c} = \frac{\mu k_F^3}{12\pi^2} - \frac{m^2\mu k_F}{8\pi^2} + \frac{m^4}{8\pi^2} \log \frac{\mu + k_F}{m} \approx \frac{\mu^4}{12\pi^2} - \frac{m^2\mu^2}{4\pi^2} + O(m^4 \log m), \quad (21)$$

where $k_F = \sqrt{\mu^2 - m^2}$ is the Fermi momentum. Substituting this equation of state in the general formula (20), one obtains the effective Lagrangian

$$\begin{aligned} \frac{1}{N_c} \mathcal{L}_{\text{eff}}(\varphi) = & \frac{1}{12\pi^2} \left[\mu^4 - 4\mu^3 \partial_0\varphi + 6\mu^2 (\partial_0\varphi)^2 - 2\mu^2 (\partial_i\varphi)^2 - 4\mu \partial_0\varphi \partial_\mu\varphi \partial^\mu\varphi + (\partial_\mu\varphi \partial^\mu\varphi)^2 \right] - \\ & - \frac{m^2}{4\pi^2} \left[\mu^2 - 2\mu \partial_0\varphi + (\partial_0\varphi)^2 - (\partial_i\varphi)^2 \right]. \end{aligned} \quad (22)$$

The bilinear terms in the Lagrangian imply that the NG boson phase velocity equals

$$v^2 = \frac{1}{3} \frac{2\mu^2 - 3m^2}{2\mu^2 - m^2} \approx \frac{1}{3} \left(1 - \frac{m^2}{\mu^2} \right) + O(m^4/\mu^4). \quad (23)$$

This can be shown to coincide, to the order displayed, with the hydrodynamic speed of sound in a free gas.

4.3. Effective Lagrangian for neutral quark matter

Let us now consider the case of several $U(1)$ symmetries with different chemical potentials. Thus, the spontaneously broken symmetry group $G = U(1)_1 \times U(1)_2 \times \dots$ is associated with the NG fields $\varphi_1, \varphi_2, \dots$, chemical potentials μ_1, μ_2, \dots , and the equation of state $P(\mu_1, \mu_2, \dots)$. Unfortunately, one can easily see that Son's trick does not work in this case. The reason is that with more fields there are more independent ways to construct a Lorentz-invariant Lagrangian density that reduces to the same function of chemical potentials for uniform fields. As an example, just observe that $D^\mu\varphi_1 D_\mu\varphi_1 D^\nu\varphi_2 D_\nu\varphi_2$ and $D^\mu\varphi_1 D^\nu\varphi_1 D_\mu\varphi_2 D_\nu\varphi_2$ both give $\mu_1^2\mu_2^2$.

Fortunately, there is a special case where Son's method can still be used. Once the equation of state separates to

$$P(\mu_1, \mu_2, \dots) = P_1(\mu_1) + P_2(\mu_2) + \dots, \quad (24)$$

the dependence of the effective action on the NG fields φ_k can again be fully reconstructed using Lorentz invariance. This generalization of the method may seem somewhat trivial, since the equation of state (24) corresponds to separate

and noninteracting subsystems. However, they can become entangled by a coupling to an additional field, as we will see later.

Let us address the specific question: what happens when a part of the symmetry group is gauged? For simplicity we will assume that there is only one gauge field that couples to a linear combination of generators of $U(1)_k$, that is, to a subgroup of G . This is determined by the charges q_k of the fields φ_k . In the effective Lagrangian we thus have to replace the combinations $\partial_\nu \varphi_k - \delta_{\nu 0} \mu_k$ with $D_\nu \varphi_k - \delta_{\nu 0} \mu_k$, where $D_\mu \varphi_k = \partial_\mu \varphi_k - eq_k A_\mu$ and e is the gauge coupling. To obtain the effective Lagrangian from the equation of state, one in turn has to replace everywhere μ_k^2 with $(D_0 \varphi_k - \mu_k)^2 - (D_i \varphi_k)^2$. Note that the effective Lagrangian also contains a term $A_0 J_0$: a coupling of the gauge field to an external background charge density which ensures that the system as a whole remains neutral despite the chemical potentials μ_k [41, 42]. This is equivalently expressed by the fact that $\langle A_0 \rangle = 0$.

We will now consider a system where the underlying equation of state is well approximated by a noninteracting Fermi gas. This is the case of color superconductors at high baryon density since the pairing effects are exponentially suppressed and the normal Fermi liquid contribution dominates the pressure. In accordance with Eq. (22), the effective Lagrangian with the leading finite-mass correction then reads $\mathcal{L}_{\text{eff}} = N_c(\mathcal{L}_0 + \mathcal{L}_1) + \mathcal{L}_g$, where (upon omitting terms of zeroth and first order in the fields)

$$\begin{aligned} \mathcal{L}_0 &= \frac{1}{12\pi^2} \sum_k \left[6\mu_k^2 (D_0 \varphi_k)^2 - 2\mu_k^2 (D_i \varphi_k)^2 - 4\mu_k D_0 \varphi_k D_\mu \varphi_k D^\mu \varphi_k + (D_\mu \varphi_k D^\mu \varphi_k)^2 \right], \\ \mathcal{L}_1 &= - \sum_k \frac{m_k^2}{4\pi^2} \left[(D_0 \varphi_k)^2 - (D_i \varphi_k)^2 \right], \quad \mathcal{L}_g = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \end{aligned} \quad (25)$$

So far we have not fixed the gauge for A_μ . Note that the covariant derivative $\partial_\mu \varphi_k - eq_k A_\mu$ is invariant under the gauge transformation, $\varphi'_k = \varphi_k + eq_k \alpha$, $A'_\mu = A_\mu + \partial_\mu \alpha$, where α is a gauge parameter. We can change variables φ_k to θ_k by $\varphi_k = R_{k\ell} \theta_\ell$, where $R_{k\ell}$ is a real square matrix such that $R_{k1} \sim q_k$ up to a common factor (the detailed form of this matrix will be specified later). The gauge transformation reads $R\theta' = R\theta + e\alpha q$ or $\theta' = \theta + e\alpha R^{-1}q$ where R, R^{-1} are matrices and θ', θ, q vectors. We can fix the gauge by choosing the gauge parameter $\alpha = -\frac{\theta_1}{e(R^{-1})_{1k}q_k}$ so that $\theta'_1 = 0$. We see that by fixing the gauge we can remove the field θ_1 , or we can set $\theta_1 = 0$. All other modes $\theta_2, \theta_3, \dots$ remain in the theory as physical fields. The resulting Lagrangian will be rather complicated, let us therefore look explicitly at least at the bilinear part of $\mathcal{L}_0 + \mathcal{L}_1$,

$$\mathcal{L}_{\text{bilin}} = \sum_k \left[\frac{2\mu_k^2 - m_k^2}{4\pi^2} (D_0 \varphi_k)^2 - \frac{2\mu_k^2 - 3m_k^2}{12\pi^2} (D_i \varphi_k)^2 \right]. \quad (26)$$

Expanding the square of the covariant derivative one obtains (no summation over k or μ is implied)

$$(D_\mu \varphi_k)^2 = (R_{k\ell} \partial_\mu \theta_\ell - eq_k A_\mu)^2 = R_{k\ell} R_{km} \partial_\mu \theta_\ell \partial_\mu \theta_m - 2eq_k R_{k\ell} A_\mu \partial_\mu \theta_\ell + e^2 q_k^2 A_\mu A_\mu. \quad (27)$$

We can see that the gauge boson acquires a mass term, as expected. Whether this affects the low-energy dynamics of the system is a matter of scales. In order to have a clear physical interpretation of the excitation spectrum, it would be better to get rid of the mixing term $A_\mu \partial_\mu \theta_\ell$. One could find the dispersion relations even with such mixing, but most likely just numerically [42]. Another, practical aspect is that once we decide to integrate out the massive gauge boson to obtain an effective Lagrangian for the NG bosons only, in the absence of the mixing this can be done perturbatively and will merely result in a modification of the NG boson interactions. On the other hand, the mixing with the gauge boson would necessarily induce corrections to the NG boson dispersion relations. Unfortunately it seems that in the most general case of Eq. (26), the mixing term cannot be removed by a judicious choice of $R_{k\ell}$ simultaneously in the temporal and spatial parts of $\mathcal{L}_{\text{bilin}}$. However, there are special cases in which the Lagrangian can be further simplified.

(i) *Zero masses.* In this case, the NG–gauge boson mixing can be removed by choosing $R_{k\ell}$ so that $\sum_k \mu_k^2 q_k R_{k\ell} = 0$ for all $\ell = 2, 3, \dots$. This means that the second and other columns of $R_{k\ell}$ should be set orthogonal to the vector $\mu_k^2 q_k$, that is, not to the linear combination that defines θ_1 . It is more convenient to define $\tilde{R}_{k\ell} = \mu_k R_{k\ell}$ since the above condition then demands that the $\ell = 2, 3, \dots$ columns of $\tilde{R}_{k\ell}$ be orthogonal to the first one. We can then choose the

whole matrix $\tilde{R}_{k\ell}$ to be orthogonal, upon which the bilinear Lagrangian (26) becomes

$$\mathcal{L}_{\text{bilin}} = \frac{1}{2\pi^2} \left\{ \sum_{\ell \neq 1} \left[(\partial_0 \theta_\ell)^2 - \frac{1}{3} (\partial_i \theta_\ell)^2 \right] + \sum_k e^2 q_k^2 \mu_k^2 \left(A_0 A_0 - \frac{1}{3} A_i A_i \right) \right\}. \quad (28)$$

The interaction terms are obtained upon expressing φ_k in terms of θ_ℓ in Eq. (25).

(ii) *Equal masses and chemical potentials.* In this (rather unphysical) case the mixing term is removed by setting $\sum_k q_k R_{k\ell} = 0$ for all $\ell = 2, 3, \dots$. We can thus choose directly the matrix $R_{k\ell}$ as orthogonal and the resulting bilinear Lagrangian reads

$$\mathcal{L}_{\text{bilin}} = \frac{2\mu^2 - m^2}{4\pi^2} \left\{ \sum_{\ell \neq 1} \left[(\partial_0 \theta_\ell)^2 - v^2 (\partial_i \theta_\ell)^2 \right] + e^2 \sum_k q_k^2 \left(A_0 A_0 - v^2 A_i A_i \right) \right\}, \quad (29)$$

where the phase velocity v is given by Eq. (23).

All the general formulas above are easily applied to the case of interest, that is, three-flavor quark matter with the electric charge neutrality constraint. Then the gauged subgroup is $U(1)_Q$, generated by the electric charge operator, $Q = (2/3, -1/3, -1/3)$. While the u and d quark masses can certainly be neglected, the strange quark mass must be taken into account, at least for the very reason that without this mass, no mismatch between the Fermi momenta of different quark flavors would arise and the ground state would be the CFL phase.

5. Conclusions

We analyzed the low-energy physics of spin-one color superconductors in terms of their soft, NG excitations. We used the Ginzburg–Landau theory to derive the excitation spectrum. As a warm-up exercise we first analyzed pairing of quarks of two flavors and a single color, which may play a role in phenomenology as a complement to the 2SC pairing. Already this simple example exhibits a phase with an unusual number of NG bosons. In particular, one of the NG bosons in the A-phase is of type-II, i.e., has a quadratic dispersion relation at low momentum, very much like the spin waves in ferromagnets. Also, thanks to the fact that a spacetime (rotational) symmetry is spontaneously broken we observed that the classification of NG modes into multiplets of unbroken symmetry holds strictly only in the long-wavelength limit. Any nonzero momentum of the soft mode further breaks the symmetry and lifts the degeneracy based on the symmetry of the ground state itself.

The main body of the paper is comprised of an investigation of a single-flavor three-color superconductor. This is the most likely candidate phase for the ground state of dense quark matter in case that strange-quark mass and electric charge neutrality effects disfavor pairing of quarks of different flavors. We have thus completed the analysis of the phase diagram started in Ref. [14] and the classification of the soft modes, sketched in our previous paper [35]. The four phases that appear in the phase diagram possess a plethora of different NG modes of the spontaneously broken color, baryon number and rotational symmetry. Those stemming from the color symmetry will eventually be absorbed into gluons, making them massive by the Anderson–Higgs mechanism. The other NG bosons will remain in the spectrum as physical soft modes.

Unlike in all the other phases, in the isotropic CSL phase all quarks can be gapped so that the NG bosons are the only truly gapless states in the spectrum. This has far-reaching consequences for the low-energy dynamics of the CSL phase such as its transport properties. We laid the foundation for a later economical calculation of the transport coefficients of the CSL phase such as the shear viscosity by deriving the low-energy effective field theory for its physical NG bosons. We used a functional technique [36] applied to the CFL phase before [40] and adapted it for the present case by introducing several independent chemical potentials and gauging a subgroup of the symmetry group, corresponding to the electric charge. The actual calculation goes beyond the scope of this paper and will be a subject of our future work.

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Appendix A. Mixing Lagrangians

Similar to what has been done in [37], let us consider the mixing Lagrangian for two fields π, H of the form

$$\mathcal{L}_{\text{mixing}} = \frac{1}{2} [(\partial_0 \pi)^2 - v^2 (\partial \pi)^2] + \frac{1}{2} [(\partial_0 H)^2 - v^2 (\partial H)^2] - \frac{1}{2} m^2 H^2 - \xi H \partial_0 \pi. \quad (\text{A.1})$$

Here v is the phase velocity; it will be sufficient to assume that it is common to both fields, even though the result is easy to generalize to the case with two different phase velocities. The field H possibly has a mass term and ξ is the mixing parameter which arises from the term in Eq. (1) with a single time derivative. The dispersion relations following from the Lagrangian (A.1) are

$$E^2 = m^2 + \xi^2 + \mathcal{O}(k^2), \quad \text{massive mode}, \quad (\text{A.2})$$

$$E^2 = \frac{m^2 v^2}{m^2 + \xi^2} k^2 + \frac{\xi^4 v^4}{(m^2 + \xi^2)^3} k^4 + \mathcal{O}(k^6), \quad \text{NG mode}. \quad (\text{A.3})$$

Note in particular that when the mass term m is zero, the mixing term ξ transforms the two expected (type-I) NG modes into one massive mode with mass $|\xi|$ and one (type-II) NG mode with the quadratic dispersion relation $E = v^2 k^2 / |\xi|$.

Appendix B. Lagrangian and dispersion relations for spin-one color superconductor

The low-energy effective Lagrangian for the superconductor is in general only constrained by rotational invariance as well as the internal symmetry of the system and, in case of QCD, conservation of parity. Its most general form for the single-flavor spin-one color superconductor therefore reads

$$\begin{aligned} \mathcal{L} = & ic_1 \text{Tr}[\Delta^\dagger \partial_0 \Delta] + c_2 \text{Tr}[(\partial_0 \Delta^\dagger)(\partial_0 \Delta)] - a_1 \text{Tr}(\partial_i \Delta \partial_i \Delta^\dagger) - a_2 (\partial_i \Delta_{ai})(\partial_j \Delta_{aj}^*) \\ & - b \text{Tr}(\Delta \Delta^\dagger) - d_1 [\text{Tr}(\Delta \Delta^\dagger)]^2 - d_2 \text{Tr}(\Delta \Delta^\dagger \Delta \Delta^\dagger) - d_3 \text{Tr}[\Delta \Delta^T (\Delta \Delta^T)^\dagger]. \end{aligned} \quad (\text{B.1})$$

In the limit that the Cooper pairs form tightly bound molecules, the ground state behaves as their Bose–Einstein condensate. The coefficients c_1 and c_2 are then related. To see this, note that adding a kinetic term to the free energy functional $\mathcal{F}[\Delta]$ can be understood as defining a Hamiltonian that governs the dynamics of the molecules, $\mathcal{H} = c_2 \text{Tr}[(\partial_0 \Delta^\dagger)(\partial_0 \Delta)] + \mathcal{F}[\Delta]$. The many-body description of the system is then accomplished by adding the chemical potential μ via $\mathcal{H} \rightarrow \mathcal{H} - \mu \mathcal{N}$ where \mathcal{N} is the Noether charge corresponding to the $U(1)_B$ symmetry (baryon number operator). Integrating out the canonical momenta in order to arrive at a Lagrangian formulation of the many-body problem [41], one finds terms with one as well as two time derivatives whose coefficients are related by $c_1 = -2\mu c_2$. Nevertheless, we will keep them as independent parameters.

Appendix B.1. CSL phase

With the knowledge of the broken and unbroken symmetry generators, we can write the order parameter Δ in the following form,

$$\Delta = \exp(i\theta) \exp\left(\frac{1}{2} i \kappa_a \lambda_a\right) (v \Delta_{\text{CSL}} + H) \exp(i v_i J_i), \quad (\text{B.2})$$

where the summations over a and i are in the ranges $a = 1, 3, 4, 6, 8$ and $i = 1, 2, 3$. The matrix field H can be parameterized by

$$H = h \mathbb{1} + \frac{1}{2} \varphi_a \lambda_a + \chi_i J_i = \begin{pmatrix} h + \frac{1}{2} \varphi_3 + \frac{1}{2\sqrt{3}} \varphi_8 & \frac{1}{2} \varphi_1 - i \chi_3 & \frac{1}{2} \varphi_4 + i \chi_2 \\ \frac{1}{2} \varphi_1 + i \chi_3 & h - \frac{1}{2} \varphi_3 + \frac{1}{2\sqrt{3}} \varphi_8 & \frac{1}{2} \varphi_6 - i \chi_1 \\ \frac{1}{2} \varphi_4 - i \chi_2 & \frac{1}{2} \varphi_6 + i \chi_1 & h - \frac{1}{\sqrt{3}} \varphi_8 \end{pmatrix}, \quad (\text{B.3})$$

where $h, \varphi_{1,3,4,6,8}, \chi_{1,2,3}$ are all massive fields. We find the following dispersion relations,

$$\begin{aligned}
E_h^2 &= \frac{-2bc_2 + c_1^2}{c_2^2} + \frac{(a_1 + a_2)(-2bc_2 + 2c_1^2)}{-2bc_2^2 + c_1^2 c_2} k^2, \\
E_\chi^2 &= \frac{-2bc_2(d_2 - d_3) + 3\bar{d}c_1^2}{3c_2^2 \bar{d}} + \frac{2(a_1 + a_2)[-bc_2(d_2 - d_3) + 3\bar{d}c_1^2]}{-2bc_2^2(d_2 - d_3) + 3\bar{d}c_1^2 c_2} k^2, \\
E_\varphi^2 &= \frac{-2bc_2(d_2 + d_3) + 3\bar{d}c_1^2}{3c_2^2 \bar{d}} + \frac{2(a_1 + a_2)[-bc_2(d_2 + d_3) + 3\bar{d}c_1^2]}{-2bc_2^2(d_2 + d_3) + 3\bar{d}c_1^2 c_2} k^2, \\
E_\theta^2 &= \frac{-2b(a_1 + a_2)}{-2bc_2 + c_1^2} k^2 + \frac{(a_1 + a_2)^2 c_1^4}{(-2bc_2 + c_1^2)^3} k^4, \\
E_\nu^2 &= \frac{-2b(a_1 + a_2)(d_2 - d_3)}{-2bc_2(d_2 - d_3) + 3\bar{d}c_1^2} k^2 + \frac{27(a_1 + a_2)^2 c_1^4 \bar{d}^3}{[-2bc_2(d_2 - d_3) + 3\bar{d}c_1^2]^3} k^4, \\
E_\kappa^2 &= \frac{-2b(a_1 + a_2)(d_2 + d_3)}{-2bc_2(d_2 + d_3) + 3\bar{d}c_1^2} k^2 + \frac{27(a_1 + a_2)^2 c_1^4 \bar{d}^3}{[-2bc_2(d_2 + d_3) + 3\bar{d}c_1^2]^3} k^4.
\end{aligned} \tag{B.4}$$

Appendix B.2. Polar phase

The order parameter can be written in the following form

$$\Delta = \exp(i\theta) \exp\left(\frac{1}{2}i\kappa_a \lambda_a\right) (v\Delta_{\text{polar}} + H) \exp(iv_i J_i), \tag{B.5}$$

where the summations over a and i are in the ranges $a = 4, 5, 6, 7$ and $i = 1, 2$. The matrix field H can be parameterized by

$$H = \begin{pmatrix} \chi_{11} + i\varphi_{11} & \chi_{12} + i\varphi_{12} & 0 \\ \chi_{21} + i\varphi_{21} & \chi_{22} + i\varphi_{22} & 0 \\ i\rho_1 & i\rho_2 & h \end{pmatrix}. \tag{B.6}$$

It comprises of two complex doublets of the unbroken $SU(2)_L$, a real vector ρ of $SO(2)_R$, and a singlet h . They are all expected to excite massive modes. The NG mode θ and the massive mode h are coupled, their eigenmodes are found to be

$$\begin{aligned}
E_h^2 &= \left(\frac{-2b}{c_2} + \frac{c_1^2}{2c_2^2}\right) + \left(1 - \frac{c_1^2}{4bc_2 - c_1^2}\right) \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2\right), \\
E_\theta^2 &= \frac{4bc_2}{4bc_2 - c_1^2} \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2\right),
\end{aligned} \tag{B.7}$$

where \perp refers to the orientation of the condensate in the spin space very much like in Sec. 2. The NG mode ν and the massive mode ρ are coupled and give the following spectrum,

$$\begin{aligned}
E_\rho^2 &= \left(\frac{m_2^2}{c_2} + \frac{c_1^2}{2c_2^2}\right) + \left(1 + \frac{c_1^2}{c_1^2 + 2c_2 m_2^2}\right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2\right), \\
E_\nu^2 &= \frac{2c_2 m_2^2}{2c_2 m_2^2 + c_1^2} \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2\right),
\end{aligned} \tag{B.8}$$

where we have defined $m_1^2 = \frac{b(d_2 + d_3)}{d}$, $m_2^2 = \frac{2bd_3}{d}$. The NG modes κ_a give

$$\begin{aligned}
E_{\kappa'}^2 &= \frac{c_1^2}{c_2^2} + 2 \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2\right), \\
E_\kappa^2 &= \left(\frac{a_1}{c_1}\right)^2 k_\perp^4 + \left(\frac{a_1 + a_2}{c_1}\right)^2 k_3^4.
\end{aligned} \tag{B.9}$$

The massive modes χ_{ij} and φ_{ij} are coupled to each other and give the spectrum

$$E_{\chi,\varphi}^2 = \left[\frac{m_1^2}{c_2} \pm \frac{c_1^2}{2c_2^2} \left(\sqrt{1 + \frac{4c_2m_1^2}{c_1^2}} \pm 1 \right) \right] + \left(1 \pm \sqrt{\frac{c_1^2}{c_1^2 + 4c_2m_1^2}} \right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right).$$

Appendix B.3. A-phase

The order parameter can be written in the following form

$$\Delta = \exp(i\theta) \exp\left(\frac{1}{2} i \kappa_a \lambda_a\right) (v \Delta_A + H) \exp(i v_i J_i), \quad (\text{B.10})$$

where the summations over a and i are in the ranges $a = 4, 5, 6, 7$ and $i = 1, 2$. The matrix H for massive fields can be parameterized as

$$H = \begin{pmatrix} \varphi_4 - i\varphi_5 & -i(\varphi_4 - i\varphi_5) & \chi_{11} + i\chi_{12} \\ \varphi_6 - i\varphi_7 & -i(\varphi_6 - i\varphi_7) & \chi_{21} + i\chi_{22} \\ h + \rho_1 + i\rho_2 & ih - i(\rho_1 + i\rho_2) & 0 \end{pmatrix}. \quad (\text{B.11})$$

The NG fields κ_4, κ_5 and the massive fields φ_4, φ_5 are coupled (in exactly the same way the NG fields κ_6, κ_7 and the massive fields φ_6, φ_7 , as they form doublets of the unbroken $\text{SU}(2)_L$) and give the dispersion relations,

$$\begin{aligned} E_{\kappa'}^2 &= \frac{c_1^2}{c_2^2} + 2 \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \\ E_\kappa^2 &= \left(\frac{a_1 + a_2}{c_1} \right)^2 k_\perp^4 + \left(\frac{a_1}{c_1} \right)^2 k_3^4, \\ E_\varphi^2 &= \frac{1}{2} \left[\frac{m_2^2}{c_2} \pm \frac{c_1^2}{2c_2^2} \left(\sqrt{1 + \frac{2c_2m_2^2}{c_1^2}} \pm 1 \right) \right] + \left(1 \pm \sqrt{\frac{c_1^2}{c_1^2 + 2c_2m_2^2}} \right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \end{aligned} \quad (\text{B.12})$$

where we have used $m_1^2 = \frac{bd_2}{d}$, $m_2^2 = \frac{2b(d_2-d_3)}{d}$, $m_3^2 = -\frac{4bd_3}{d}$. The NG mode θ and the massive modes h, ρ are coupled and give the eigenmodes,

$$\begin{aligned} E_\theta^2 &= \left(1 + \frac{c_1^2}{2bc_2 - c_1^2} \right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \\ E_h^2 &= \frac{-2b}{c_2} + \frac{c_1^2}{c_2^2} + \left(1 - \frac{c_1^2}{2bc_2 - c_1^2} \right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \\ E_\rho^2 &= \frac{m_3^2}{2c_2} + \frac{c_1^2}{2c_2^2} \left[1 \pm \sqrt{1 + \frac{(c_2m_3^2 + 4bc_2)^2 + 4c_1^2c_2m_3^2}{4c_1^4}} \right] \\ &\quad + \left[1 \pm \sqrt{\frac{4c_1^4}{(c_2m_3^2 + 4bc_2)^2 + 4c_1^2c_2m_3^2 + 4c_1^4}} \right] \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right). \end{aligned}$$

The NG modes $v_{1,2}$ give

$$\begin{aligned} E_{v'}^2 &= \frac{c_1^2}{c_2^2} + 2 \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2 \right), \\ E_v^2 &= \left(\frac{a_1}{c_1} \right)^2 k_\perp^4 + \left(\frac{a_1 + a_2}{c_1} \right)^2 k_3^4. \end{aligned} \quad (\text{B.13})$$

The massive modes χ_{ij} ($i, j = 1, 2$) give

$$E_\chi^2 = \left[\frac{m_1^2}{c_2} \pm \frac{c_1^2}{2c_2^2} \left(\sqrt{1 + \frac{4c_2m_1^2}{c_1^2}} \pm 1 \right) \right] + \left(1 \pm \sqrt{\frac{c_1^2}{c_1^2 + 4c_2m_1^2}} \right) \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2 \right).$$

Appendix B.4. ε -phase

The order parameter can be written in the following form

$$\Delta = \exp(i\theta_2 \mathcal{P}_2) \exp(i\theta_3 \mathcal{P}_3) \exp\left(i\frac{1}{2}\kappa_a \lambda_a\right) (v\Delta_\varepsilon + H) \exp(iv_i J_i) \quad (\text{B.14})$$

where the summations over a and i are in the ranges $a = 1, 2, 4, 5, 6, 7$ and $i = 1, 2$. The matrix H for massive fields can be parameterized as

$$H = \begin{pmatrix} \varphi_4 - i\varphi_5 & -i(\varphi_4 - i\varphi_5) & 0 \\ \varphi_6 - i\varphi_7 & -i(\varphi_6 - i\varphi_7) & h_2 \\ h_3 + \rho_1 + i\rho_2 & ih_3 - i(\rho_1 + i\rho_2) & 0 \end{pmatrix}. \quad (\text{B.15})$$

We can define masses

$$\begin{aligned} m_2^2 &= -\frac{2b[d_1 d_2 + d_2(d_2 + d_3)]}{\bar{d}(2d_2 + d_3)}, \quad m_3^2 = -\frac{4b(d_1 + d_2)(d_2 + d_3)}{\bar{d}(2d_2 + d_3)}, \\ m_{23}^2 &= -\frac{4\sqrt{2}bd_1\sqrt{\bar{d}_2(d_2 + d_3)}}{\bar{d}(2d_2 + d_3)}, \quad m_{45}^2 = -\frac{2b(d_3^2 - d_2^2)}{\bar{d}(2d_2 + d_3)}, \\ m_{67}^2 &= -\frac{2bd_3^2}{\bar{d}(2d_2 + d_3)}, \quad m_\rho^2 = -\frac{4bd_2(d_2 + d_3)}{\bar{d}(2d_2 + d_3)}. \end{aligned}$$

The NG modes θ_2, θ_3 and the massive modes h_2, h_3, ρ_1, ρ_2 are coupled and give the following eigenmodes,

$$\begin{aligned} E_{\theta_2}^2 &= \frac{m_{23}^2 + 4m_2^2}{m_{23}^2 + 4m_2^2 + 4c_1^2/c_2^2} \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2 \right), \\ E_h^2 &= \frac{m_{23}^2 + 4m_2^2}{4c_2} + \frac{c_1^2}{c_2^2} + \left(1 + \frac{4c_1^2/c_2^2}{m_{23}^2 + 4m_2^2 + 4c_1^2/c_2^2} \right) \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2 \right), \\ E_{\theta_3}^2 &= \frac{m_3^2 + m_{23}^2}{m_3^2 + m_{23}^2 + 2c_1^2/c_2} \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \\ E_h^2 &= \frac{m_3^2 + m_{23}^2}{4c_2} + \frac{c_1^2}{c_2^2} + \left(1 + \frac{2c_1^2/c_2}{m_3^2 + m_{23}^2 + 2c_1^2/c_2} \right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \\ E_\rho^2 &= \frac{m_\rho^2}{2c_2} + \frac{c_1^2}{2c_2^2} \left(1 \pm \sqrt{1 + \frac{2c_2 m_\rho^2}{c_1^2}} \right) + \left(1 \pm \sqrt{\frac{c_1^2}{c_1^2 + 2c_2 m_\rho^2}} \right) \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right). \end{aligned}$$

For NG bosons $\kappa_{1,2}$ we obtain the energies,

$$\begin{aligned} E_\kappa^2 &= \frac{c_1^2}{c_2^2} + 2 \left(\frac{a_1}{c_2} k_\perp^2 + \frac{a_1 + a_2}{c_2} k_3^2 \right), \\ E_{\kappa'}^2 &= \left(\frac{a_1}{c_1} \right)^2 k_\perp^4 + \left(\frac{a_1 + a_2}{c_1} \right)^2 k_3^4. \end{aligned} \quad (\text{B.16})$$

The NG modes $\kappa_{4,5}$ and the massive modes $\varphi_{4,5}$ are coupled (in the same way as $\kappa_{6,7}$ are coupled with $\varphi_{6,7}$) and give the dispersion relations

$$\begin{aligned} E_\kappa^2 &= \frac{c_1^2}{c_2^2} + 2 \left(\frac{a_1 + a_2}{c_2} k_\perp^2 + \frac{a_1}{c_2} k_3^2 \right), \\ E_{\kappa'}^2 &= \left(\frac{a_1 + a_2}{c_1} \right)^2 k_\perp^4 + \left(\frac{a_1}{c_1} \right)^2 k_3^4, \\ E_\varphi^2 &= \frac{m_{45}^2}{c_2} + \frac{c_1^2}{2c_2^2} \left(1 \pm \sqrt{1 + \frac{2c_2 m_{45}^2}{c_1^2}} \right) + 1 \pm \sqrt{\frac{c_1^2}{c_1^2 + 2c_2 m_{45}^2}}. \end{aligned} \quad (\text{B.17})$$

The NG modes $\nu_{1,2}$ give

$$\begin{aligned} E_{\nu'}^2 &= \frac{c_1^2}{c_2^2} + 2 \left(\frac{a_1}{c_2} k_{\perp}^2 + \frac{a_1 + a_2}{c_2} k_3^2 \right), \\ E_{\nu}^2 &= \left(\frac{a_1}{c_1} \right)^2 k_{\perp}^4 + \left(\frac{a_1 + a_2}{c_1} \right)^2 k_3^4. \end{aligned} \quad (\text{B.18})$$

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